Complexity of Reconfiguring Vertex-Disjoint Shortest Paths*

Rin Saito Takehiro Ito

rin.saito@dc.tohoku.ac.jp takehiro@tohoku.ac.jp

Graduate School of Information Sciences, Tohoku University, Sendai, Japan

Combinatorial reconfiguration [2] has been extensively studied in the field of theoretical computer science. We study the complexity of the *Reachability of Vertex-Disjoint Shortest Paths* (RVDSP) problem, defined as follows: Given two tuples of internally vertex-disjoint shortest paths for k (≥ 1) prescribed terminal pairs in an unweighted graph, RVDSP asks whether there exists a step-by-step transformation from one tuple to the other by exchanging a single vertex of one shortest path in the current tuple at a time, so that the intermediate results maintain tuples of internally vertex-disjoint shortest paths. Originally, RVDSP for $k = 1$ have been studied as the Shortest Path Reconfiguration (SPR) problem [1, 2]. Saito et al. [3] introduced RVDSP, and showed that it is PSPACE-complete for any fixed k, even on bipartite graphs.

We first analyze the complexity of RVDSP from the viewpoint of the maximum terminal distance, which is the maximum distance between any terminal pair.

Theorem 1. RVDSP *is solvable in polynomial time when the maximum terminal distance is at most two, while it is* PSPACE-*complete when the maximum terminal distance is c for any fixed integer $c \geq 3$.*

We note that our analysis is tight with respect to the maximum terminal distance. As a byproduct, our proof for the above hardness result also yields that RVDSP remains PSPACE-complete even for planar bipartite graphs of bounded bandwidth with constant maximum degree, where the maximum terminal distance is at least four.

We next study the parameterized complexity of RVDSP. We focus on the number ℓ of steps for transforming two given tuples of internally vertex-disjoint shortest paths. Bousquet et al. [1] showed that SPR (i.e., RVDSP for $k = 1$) for nowhere-dense classes of graphs is fixed-parameter tractable when parameterized by ℓ. By naturally extending this result, we establish that RVDSP for such classes is fixed-parameter tractable when parameterized by $\ell + k$. In addition, instead of k, we consider the maximum degree Δ of the input graph; recall that RVDSP remains PSPACE-complete even for graphs of bounded maximum degree.

Theorem 2. RVDSP *is fixed-parameter tractable when parameterized by $\ell + \Delta$.*

Finally, we analyze RVDSP from the viewpoint of graph width parameters. Bousquet et al. [1] showed that SPR is fixed-parameter tractable when parameterized by modular width mw, cluster deletion number cd, or treedepth td of the input graph. By taking k as an additional parameter, we extend these results for SPR to RVDSP, as follows.

Theorem 3. RVDSP *is fixed-parameter tractable when parameterized by $\mathsf{mw} + k$, $\mathsf{cd} + k$, or $\mathsf{td} + k$.*

References

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