Online Exploration of Grid Graphs with Multiple Searchers

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1 Introduction

We study the problem of online graph exploration by multiple searchers. The problem is motivated by the autonomous exploration and mining of extraterrestrial resources and the need to efficiently explore the inside of buildings or underground shopping areas in disaster situations. The problem is formally defined as follows: Given an unknown graph $G = (V, E, s)$, k agents are initially located at the vertex $s \in V$. At each discrete time step, each agent can choose to move to an adjacent vertex or stay at the current vertex. Initially, agents know only the information of the edges incident to s, and they can obtain the information of the edges incident to a vertex when they newly visit it (hence the term "online"). The goal of the online graph exploration problem is to minimize the time steps until all vertices are visited by some agents and all agents return to s.

We focus on online graph exploration, where the input graph is a *grid graph*, defined as follows. A grid graph is a finite, connected induced subgraph of the infinite two-dimensional grid graph, where the infinite grid graph is defined as the graph whose vertex set is $\mathbb{Z}^2$ and in which two vertices are adjacent if and only if their Euclidean distance is 1 (i.e. they differ by exactly 1 in one coordinate and are equal in the other). Considering online exploration in grid graphs is both natural and useful. This is because real-world exploration regions can be modeled as grid graphs by virtually constructing a grid based on the agent's visibility distance, treating each cell as a vertex, and defining edges between adjacent cells.

In this paper, we present the first algorithm for online exploration in grid graphs whose competitive ratio is independent of the size of the input graph.

2 Related Works

In the context of online graph exploration by multiple searchers, research has been conducted on trees, cycles [1], and tadpole graphs [2]. On the other hand, studies on more general graph classes are extremely limited. To the best of our knowledge, the only existing work in this direction is that of Ortolf and Schindelhauer [3], who studied exploration of $n \times n$ square grid graphs with multiple rectangular holes. They proposed an online exploration algorithm with a competitive ratio of $\mathcal{O}(\log^2 n)$ and established a lower bound of $\Omega(\log k/ \log\log k)$ for arbitrary deterministic algorithms.

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3 Results

Our proposed algorithm, GTE (Grid exploration via Tree Embedding), extends the tree exploration algorithm proposed by Higashikawa et al. [1], called BEER. The GTE algorithm is an online algorithm that incrementally embeds a spanning tree into an input grid graph at newly visited vertices and applies BEER to explore the locally embedded tree, thereby progressing the exploration.

Here, we provide definitions of the terms necessary to describe our results. We define a *hole* as a bounded face of a grid graph with an area of at least 2, and a *corner* of a hole as a vertex corresponding to a convex angle of a hole. The following is the main lemma of the paper.

Lemma 1. Let $D(G)$ be the distance from s to the farthest vertex in G. Then, for any grid graph G with c corners of holes, it holds that

$$D(T_{\mathrm{GTE}}(G)) \leq (2c + 1)D(G),$$

where $T_{\mathrm{GTE}}(G)$ is the tree embedded in G by the GTE algorithm.

We combine Lemma 1 with the analysis of the exploration cost of BEER [1], which leads to the following theorem.

Theorem 1. For any grid graph with c corners of holes, the GTE algorithm with k searchers achieves a competitive ratio of $\mathcal{O}((c + 1)k/\log k)$.

Regarding a lower bound on the competitive ratio, we prove the following result, which implies that for grid graphs with a constant number of corners of holes, our algorithm achieves an asymptotically tight competitive ratio among so-called greedy algorithms.

Theorem 2. For any $\delta > 0$, no greedy online algorithm for grid graph with k searchers achieves a competitive ratio less than $\lceil k/(1 + \lfloor \log k \rfloor)\rceil - \delta$.

References

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