Space-Efficient Data Structure for $(k \dots 21)$-avoiding Involutions

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A permutation $\pi\colon [n] \to [n]$ is said to be $(k \dots 21)$-avoiding if π does not contain a decreasing subsequence of length k. Moreover, π is called an involution if $\pi = \pi^{-1}$. In this paper, we consider the design of a space-efficient data structure for a $(k \dots 21)$-avoiding involution π that supports $\pi(i)$ queries in sub-linear time.

For general permutations, the information-theoretical lower bound is $n \log n - O(n)$ bits[1]. $(k \dots 21)$-avoiding permutations and $(k \dots 21)$-avoiding involutions are among subclasses of general permutation whose information-theoretical lower bound is $O(n)$ bits. It follows from Regev's result [3] that when $k = O(1)$, the information-theoretical lower bound for these classes are $2n \log(k-1) - o(n)$ bits and $n \log(k-1) - o(n)$ bits, respectively.

Barbay et al. [1,2] showed that there exists a data structure for a $(k \dots 21)$-avoiding permutation π of length n that uses $2n \log(k-1) + o(n)$ bits and supports $\pi(i)$ and $\pi^{-1}(j)$ queries, which is succinct when $k = O(1)$. The key idea of their structure is that any $(k \dots 21)$-avoiding permutation can be decomposed into at most $k-1$ increasing subsequences. Building on this idea, we propose a data structure for a $(k \dots 21)$-avoiding permutation π of length n that uses $n \log(k-1) + o(n)$ bits if k is even and $n \log k + o(n)$ bits if k is odd. This data structure is succinct when k is an even constant. Table 1 summarizes space and time complexity of data structures for some subclasses of permutations.

Table 1: Data structures for some subclasses of permutations

Reference	Class	Information -theoretical lower bound [2]	Space [2]	Query time of $\pi(i)$ and $\pi^{-1}(j)$
[1, 2]	$(k \dots 21)$-avoiding permutation	$2n \log(k-1)$ $- o(n)$ bits	succinct	$O\left(1 + \frac{\log k}{\log \log n}\right)$
New	$(k \dots 21)$**-avoiding involution** (k is even)	$n \log(k-1)$ $- o(n)$ bits	succinct	$O\left(1 + \frac{\log k}{\log \log n}\right)$
New	$(k \dots 21)$**-avoiding involution** (k is odd)	$n \log(k-1)$ $- o(n)$ bits	$n \log k$ $+ o(n)$ bits	$O\left(1 + \frac{\log k}{\log \log n}\right)$

References

[1] Jérémy Barbay and Gonzalo Navarro. On compressing permutations and adaptive sorting. *Theoretical Computer Science*, 513:109–123, 2013.

[2] Jérémy Barbay, Francisco Claude Faust, Travis Gagie, Gonzalo Navarro, and Yakov Nekrich. Efficient fully-compressed sequence representations. *Algorithmica*, 69, 11 2009.

[3] Amitai Regev. Asymptotic values for degrees associated with strips of Young diagrams. *Advances in Mathematics*, 41(2):115–136, 1981.

[1] In this paper, log denotes the logarithm to the base 2.

[2] Information-theoretical lower bounds and space succinctness are considered under $k = O(1)$.