Learning-Augmented Algorithms for Online Time-Window TSP

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We consider the online version of the time-window traveling salesman problem (OTWTSP), a variation of the classic traveling salesman problem (TSP) with time-window constraints (TWTSP) on input requests. In this problem, a salesman starting at an arbitrary point in a metric space is required to serve online requests, where each request requires a unit of service time, arriving at its release time, associated with a specific deadline after which the request expires. The online nature indicates that the salesman has no information about a request's position and deadline before its release time. Consequently, some requests may expire before they can be served, which increases the problem complexity. The objective of the problem is to find a route that maximizes the number of requests served, or the amount of profits if each request is associated with a nonuniform profit.

Due to the time-window constraint, there have been only few studies on OTWTSP in the literature. To the best of our knowledge, this problem has been considered by the following works. Azar and Vardi first introduced online algorithms with a constant competitive ratio for the OTWTSP under the assumption that the time-windows are significantly larger than the diameter of a given graph [1]. Recently, Chawla and Christou proposed a learning-augmented algorithm given the predictions of the positions and the time-windows of the online requests [2]. Their algorithm achieves a competitive ratio logarithmic in the diameter of the graph and polynomial in the prediction error, specifically, the location error. However, this result only applies to the situations in which the location error and the time-window error are small. That is, it guarantees only the consistency of the algorithm, i.e., performing well with sufficiently good predictions. It would be worthwhile to study learning-augmented algorithms with robustness guarantees.

In this study, we plan to incorporate a different approach to designing a prediction strategy. In particular, we first introduce a schedule-based algorithm that integrates a waiting strategy based on the worthiness of the offline deadline-TSP route obtained by Bansal et al. [3]. To be precise, instead of predicting the information of each input request, we incorporate the prediction regarding whether we should start following the route at a proper moment. The strategy will allow the algorithm not to lead to a competitive ratio being sensitive to the prediction errors. It is expected that the waiting strategy could help derive an online algorithm with a better competitive ratio with robust guarantees.

Keywords: learning-augmented algorithm, online algorithm, competitive ratio, traveling salesman

problem

References:

[1] Yossi Azar and Adi Vardi. 2015. TSP with Time Windows and Service Time. arXiv preprint arXiv:1501.06158 (2015).

[2] Shuchi Chawla and Dimitris Christou. 2024. Online Time-Windows TSP with Predictions. In Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques (APPROX/RANDOM 2024), Amit Kumar and Noga Ron-Zewi (Eds.). Leibniz International Proceedings in Informatics (LIPIcs), Vol. 317, 2:1–2:21. https://doi.org/10.4230/LIPIcs.APPROX/RANDOM.2024.2

[3] Nikhil Bansal, Avrim Blum, Shuchi Chawla, and Adam Meyerson. 2004. Approximation algorithms for deadline-TSP and vehicle routing with time-windows. In Proceedings of the Thirty-Sixth Annual ACM Symposium on Theory of Computing (STOC '04). ACM, New York, NY, USA, 166–174. https://doi.org/10.1145/1007352.1007385