Yavalath is PSPACE-Complete

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1 Introduction

Yavalath is an abstract board game played on a hexagonal grid designed by an AI program [1]. In this paper, we consider two-player Yavalath played on a game board of an arbitrary size. We prove that it is PSPACE-complete to decide the winner from the given situation. We also consider modifying Yavalath by changing the number of stones in the winning or losing conditions. We prove PSPACE-completeness for modified Yavalath on the game board of an arbitrary shape under some winning and losing conditions.

2 Rules of Yavalath

Yavalath can be played by two or three players. We show only the rules of the two-player game. The game board is a hexagonal grid with five cells on each side as shown in Fig.1. One player has white stones and the other player has black stones. Starting from the blank game board, each player takes turns placing a stone of their color on an empty cell. Only on the first move of the second player, he can exchange the colors of stones. A player wins when a line of four or more stones of his color is made. A player loses when a line of three stones of his color is made without making a line of four or more stones at the same time. The game is a draw if the board fills up without satisfying the winning or losing condition.

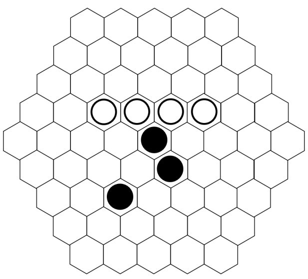

Figure 1: Game board of Yavalath.

3 PSPACE-completeness

In this section, we consider the problem to decide if the player on his turn can win the game starting from the given situation.

Theorem 1 Yavalath is PSPACE-complete.

It is obvious that the problem is in PSPACE. PSPACE-hardness is shown by reduction from generalized geography on directed planar bipartite graphs with maximum degree three [2]. The constructed game board consists of parallelogram tiles that represent edges (straight or bent), vertices (in-degree one or in-degree-two for both players), or blank spaces. Each player alternately makes a structure in which he can win in two turns and obstruct that of the other player. In addition, there exists a structure a player can win in three turns for both players, using which he can win when he wins in generalized geography.

4 Modified Yavalath

In this section, we consider the modified Yavalath such that the number of stones in a line on the winning and losing condition are different from the original Yavalath. A player wins when a line of k or more stones of his color is made. A player loses when a line of exactly either of $l_1, l_2, \ldots, l_t$ $(l_1 < l_2 < \cdots < l_t < k)$ stones of his color is made without satisfying the winning condition. We assume that the game board is of an arbitrary shape. That is, the game board need not be a regular hexagon and may have holes.

Theorem 2 Modified Yavalath on an arbitrary game board with $k \geq 4,\ t \geq 1,\ \lceil \frac{k}{2} \rceil < l_1 < l_2 < \cdots < l_t < k)$ is PSPACE-complete.

Acknowledgments

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References

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[2] D. Lichtenstein and M. Sipser, "Go is polynomial–space hard," Journal of the ACM, vol.27, no.2, 393-401, 1980.