On the Efficiency of Fair and Truthful Trade Mechanisms

The field of *Mechanism Design* studies the design of mechanisms that obtain good outcomes (as high social welfare or revenue) in the presence of strategic traders. In this paper, we consider imposing a fairness requirement on truthful trade mechanisms, and the impact of such a requirement on the economic efficiency (social welfare) of the mechanism. That is, a mechanism results in trade which generates gains, and (some of these) gains are allocated to the traders. We take a normative approach and search for simple truthful mechanisms that are constrained to distribute these gains "fairly", studying their economic efficiency. This problem can be viewed as searching for a fair solution to the bargaining problem between the two traders, but in settings with private valuations. Thus, for trade settings, our work focuses on *the implications of the combination of strategic behavior and fairness requirements on the efficiency of the outcome.*

We focus on the fundamental mechanism-design problem of bilateral trade in the Bayesian setting (Myerson and Satterthwaite, 1983), that is, a seller that sells a single good to a buyer, where traders' values for the good are private, but are sampled independently from a known Bayesian prior. The most basic of these problems is the setting where the seller has no value for the good. We call this basic setting the *zero-value seller setting* (Myerson, 1981). In spirit, the bilateral trade problem is a cooperative bargaining problem (Nash, 1951), where the traders need to ex ante agree on a truthful mechanism that is fair. If they fail to reach an agreement, the default outcome of no trade occurs. Yet, the set over which the traders are bargaining is not explicitly given, but rather induced by the traders' value distributions. One prominent solution to cooperative bargaining problems is the Kalai-Smorodinsky (KS) solution (Kalai and Smorodinsky, 1975).

Our work mainly focuses on studying the GFT of mechanisms that are *KS-fair*, satisfying the "Kalai-Smorodinsky condition": mechanisms that ex ante equalize the fraction of the ideal utilities of the two traders. Although the Kalai-Smorodinsky solution satisfies KS-fairness, the mechanism corresponding to that solution may be complex, making both theoretical analysis and practical implementation challenging. This motivates us to design *simple* mechanisms that are KS-fair and guarantee good GFT. Therefore, we pose the following question: *How large is the fraction of the* Second-Best Benchmark *that can be guaranteed by a simple mechanism satisfying KS-fairness?*

In the following we present simple mechanisms that are KS-fair and give nearly the best, or almost the best, fraction of the Second-Best Benchmark we can hope for from any KS-fair mechanism (and split all the gains between the two traders). Furthermore, while these mechanisms may not always be Pareto-optimal (with the Pareto-optimal solution being the Kalai-Smorodinsky solution), they imply that the KS solution is also guaranteed to obtain at least the same GFT approximation.

Our contributions. As the first result of this work, we establish that the optimal GFT approximation of truthful mechanisms under KS-fairness is 50%.

Theorem 1. *For every bilateral trade instance (i.e., any pair of seller and buyer distributions), there exists a truthful mechanism (λ-*Biased Random Offer Mechanism*) that is KS-fair and guarantees a GFT of at least 50% of the* Second-Best Benchmark. *Moreover, for any $C > 50\%$, there exists a zero-value seller instance in which no KS-fair truthful mechanism can achieve a C-fraction of the* Second-Best Benchmark.

To prove above theorem, we develop a black-box reduction that converts any mechanism (possibly not KS-fair) into a KS-fair mechanism whose GFT is at least C-fraction of the sum of the traders' ideal utilities, where C is the smaller ratio between each trader's ex ante utility in the original mechanism, and her own ideal utility. Our black-box reduction framework is both simple and general, and thus might be of independent interest. We also study the bilateral trade instances where both traders' distributions satisfy the monotone hazard rate (MHR) condition. In this setting, we prove a stronger guarantee of $\frac{1}{e-1} \geq 58.1\%$ for the KS-fair λ-Biased Random Offer Mechanism

In the second part of this work, we focus on zero-value seller instances where the buyer's value distribution is either regular, or satisfy the monotone hazard rate (MHR) condition.

Theorem 2. *For every zero-value seller instance where the buyer has a regular (resp. MHR) distribution, there exists a* Fixed Price Mechanism *(which is truthful) that is KS-fair, and whose GFT is at least* 85.1% *(resp.* 91.3%*) of the* Second-Best Benchmark. *Moreover, there exists a zero-value seller instance in which the buyer has a regular (resp. MHR) distribution and no KS-fair truthful mechanism obtains more than* 87.7% *(resp.* 94.4%*) of the* Second-Best Benchmark.

Besides KS-fairness, we also explore alternative fairness definitions for the bilateral trade model. In particular, we obtain the optimal GFT approximation bounds of 50% and 0% for the Nash solution (Nash, 1951) (i.e., NSW-Maximizing Mechanism) and the egalitarian solution (Kalai, 1977) (i.e., equitable mechanisms), respectively.

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