Guarding Terrains with Guards on a Line

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The terrain guarding problem is a well-known problem in the field of computational geometry. In this problem, we are given an x-monotone polygonal chain T (terrain), an integer k, and a horizontal line L lying above T, and we aim to place k point guards that together cover T. We call the problem of finding the lowest such L the *altitude terrain cover* problem. See Figure 1(a) for an illustration with $k = 2$.

We also study a variant with an additional requirement that T is partitioned into k subchains so that each subchain is paired with exactly one guard and every point on a subchain is visible from its paired guard. We call it the *bijective altitude terrain cover problem.* We consider two cases. (1) Given a line L, find a minimum-sized set of guards covering T. (2) Given an integer k, find the lowest L. See Figure 1(b) for an illustration with $k = 2$.

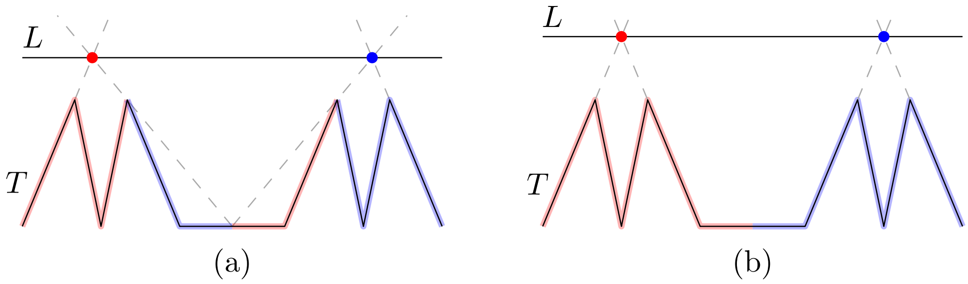

Figure 1: (a) The lowest line L such that T can be covered by two guards on L. The red subchains are covered by the red guard and the blue subchains are covered by the blue guard. (b) The lowest line L such that T is partitioned into two subchains. One is visible from the red guard and the other is visible from the blue guard.

Previous works. Daescu et al. studied the problem of placing the minimum number of point guards on a fixed line so that every point on the terrain is visible from some guard. They presented a linear-time algorithm for the problem and showed that the problem is NP-hard for a polyhedral terrain.

Our contribution. For the altitude terrain cover problem, we give an algorithm with running time $O(k^2\lambda_{k-1}(n)\log n)$ if k is even, and $O(k^2\lambda_{k-2}(n)\log n)$ when $k \geqslant 3$ is odd, where k is the number of guards and $\lambda_s(n)$ is the maximum possible length of an (n, s)-Davenport-Schinzel sequence. Thus, for small k, our algorithm runs in near-linear time.

For the bijective altitude terrain cover problem, we give a linear time algorithm for the case (1), and two $O(kn)$-time algorithms for the case (2) (one for even k and the other for odd k).

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