The Public University Secretary Problem with Predictions

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In recent years, the rapid development of learning predictions has brought convenience and efficiency to many research fields, including online algorithms. Machine learning can be leveraged to enhance the performance of online algorithms, which often face the challenge of making appropriate decisions with limited information about the future. By providing predictions, learning models help compensate for the lack of future information, thereby improving the performance of online algorithms. However, predictions may contain large errors, leading to significant performance degradation. In this study, we consider a variant of the well-known *k*-secretary problem and attempt to achieve better outcomes by utilizing learning predictions to assist in solving this problem while ensuring robustness to inaccurate predictions.

The classical *k*-secretary problem [1] assumes that there are n candidates, and the manager wants to select the best k secretaries from them in an online fashion. Each candidate is associated with a different quality or ranking, but the manager can only interview them sequentially and must decide immediately whether to hire or reject a candidate after each interview. Note that only one candidate can be selected at a time, and once a candidate is rejected, they cannot be reconsidered. Recently, Fujii and Yoshida revisited the famous problem and first incorporated predictions into the problem in 2024 [2].

In this study we investigate a variation of the *k*-secretary problem, i.e., the public university secretary problem, initialized by Moseley, Newman, and Pruhs in 2024 [3] using the learning-augmented mechanism. The key difference between the public secretary problem and the classical k-secretary problem is the objective of the public secretary problem, aiming to hire k cheapest secretaries instead of the secretaries with most value under a given budget. That is, assume an online algorithm knows a priori of the upper bound of the optimal solution, i.e. budget B.

Our problem setting is based on [3], but with a prediction scenario. Suppose an

online algorithm can predict the k-th smallest cost among all secretaries, denoted by p, given the number of secretary candidates, i.e. n, whereas in the original setting, n is not necessary to know. The goal is to hire a set of k secretaries such that their total cost is minimized. We incorporate predictions into the deterministic algorithms by [2] and derive a better consistency ratio if the prediction error is not large. We also analyze its robustness, ensuring that even under large prediction errors in the fully adversarial setting, the algorithm can still maintain an acceptable competitive performance.

Keywords: k-secretary, public university secretary, learning augmented, competitive analysis

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