## Fair Allocation of Items in Multiple Regions

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## Abstract

We initiate the study of fair allocation with the set of divisible or indivisible items distributed in multiple regions. The key requirement is that each agent can only obtain items from one region. In this work, we consider two kinds of fairness concepts: envy-based notions including *envy-freeness* (EF) and *envy-freeness up to one/any item* (EF1/EFX), and share-based notions including *proportionality* (PROP) and *proportionality up to one/any item* (PROP1/PROPX). On the negative side, we show NP-hardness and inapproximability results about the aforementioned fairness notions. On the positive side, we propose several algorithms to compute the partial allocations that satisfy envy-based notions and allocations that approximate the above fairness notions.

Fair allocation studies how to *fairly* allocate a set of items (resources) among agents, which has attracted the attention of many scholars in both theoretical and industrial fields. We observe that there are many real-life scenarios, where the items are distributed in multiple regions, and each agent can only obtain the items from a single region. For example, let us consider a scenario where a multinational corporation with many branches in different countries and regions wants to recruit employees. The manager not only assigns employees who meet the requirements to different regions but also allocates different items such as wages, housing, insurance, and medical care to them. Different agents have different valuations for those items, e.g., some employees prefer to live in low-floor dormitories because it is convenient, while others prefer to live in high-floor dormitories because it is quiet. Such scenarios raise a natural question. How to allocate the items to a set of agents where each agent can only obtain items from one region?

A simple way is to allocate those items equally to the employees in the same region. One difficulty is that it may be impossible to divide every item equally, e.g., every dormitory is unique because of the building and the level it belongs to. One may also wonder how about if we divide the whole allocation procedure into two steps: (1) assign agents to different regions based on their preferences; (2) use existing algorithms in fair allocation to allocate items to agents in each region. The main drawback of this approach is that assigning agents to the regions only on their preferences may cause some crowded regions, which means that every agent in this region may get a low value. The situation may worsen, especially when agents assigned to the same region have similar preferences over items.

We develop a formal model for fair allocation in multiple regions. Our model is flexible enough to capture both (1) settings where items are divisible and (2) settings where items are indivisible. Further, it can handle both (a) diverse-region case: every agent has different total valuations for different regions, and (b) equal-region case: every agent has the same total valuation for different regions. When the items are *divisible*, we study fairness criteria including *envy-freeness* (EF), EF-with-bounded-charity, and *proportionality* (PROP). When the items are *indivisible*, we are interested in the fairness notions including *envy-freeness up to one/any item* (EF1/EFX), EF1/EFX-with-bounded-charity, and *proportionality up to one/any item* (PROP1/PROPX). For each notion, we give an almost complete result. The full paper can be found at https://ojs.aaai.org/index.php/AAAI/article/view/28861.