On the Subsidy of Envy-Free Orientations in Graphs*

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Abstract

We study a fair division problem in (multi)graphs where n agents (vertices) are pairwise connected by items (edges), and each agent is interested in its incident items. We consider how to allocate items to incident agents in an envy-free manner, i.e., envy-free orientations, while minimizing the overall payment, i.e., subsidy. We first prove that computing an envy-free orientation with the minimum subsidy is NP-hard, even when the graph is simple and the agents have bi-valued additive valuations. We then bound the worst-case subsidy. We prove that for any multigraph (i.e., allowing parallel edges) and monotone valuations where the marginal value of each good is at most \$1 for each agent, \$1 each (a total subsidy of $n-1$, where n is the number of agents) is sufficient. This is one of the few cases where linear subsidy $\Theta(n)$ is known to be necessary and sufficient to guarantee envy-freeness when agents have monotone valuations. When the valuations are additive (while the graph may contain parallel edges) and when the graph is simple (while the valuations may be monotone), we improve the bound to $n/2$ and $n-2$, respectively. Moreover, these two bounds are tight.

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