Monotone Partitions of Simple Polygons

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We study the problem of partitioning a simple polygon into x-monotone or y-monotone subpolygons, using horizontal line segments (diagonals) each connecting two points on the boundary of the polygon. The objective is to minimize the size of the partition. We call this the **minimum monotone partition problem**. We study three versions of the problem: (1) every subpolygon is x-monotone, (2) every subpolygon is y-monotone, and (3) every subpolygon is x- or y-monotone. See Figure 1.

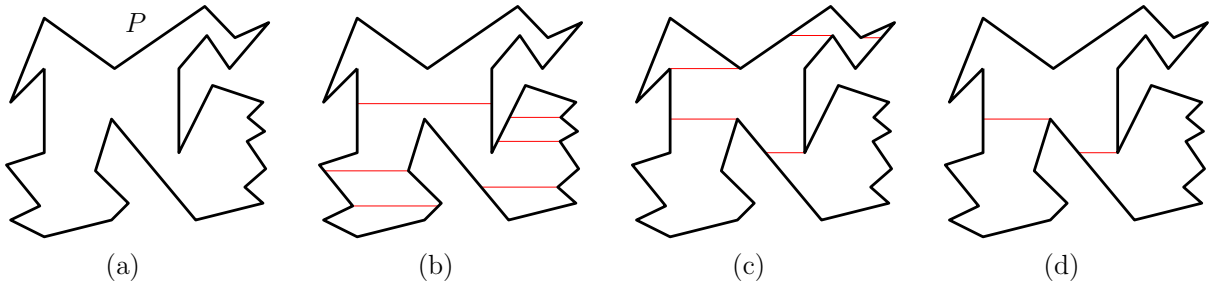

Figure 1: (a) A simple polygon P. (b) Optimal x-monotone partition of P. (c) Optimal y-monotone partition of P. (d) Optimal x- or y-monotone partition of P.

Previous works. There has been a fair amount of research on monotone partitioning of a simple polygon, using line segments connecting two distinct vertices of the polygon. The most closely related one to our research is the work by Liu. Given a rectilinear polygon with rectilinear holes with n vertices in total, his algorithm partitions the polygon into minimum number of x-monotone subpolygons using horizontal line segments, and runs in $O(n \log n)$ time.

Our contribution. We give optimal, linear time algorithms for all three versions of the minimum monotone partition problem when the input is a simple polygon. Our algorithm for version (1) improves upon the previously best $O(n \log n)$-time algorithm by Liu. We also show that version (3) (every subpolygon is x- or y-monotone) is NP-complete if holes are allowed, by a reduction from the planar 3-SAT problem. It is in stark contrast with the polynomial-time solvability of the other two versions.

For the problem of finding an orientation θ which minimizes the size of an optimal partition of the polygon into θ- or $(\theta + \pi/2)$-monotone subpolygons using parallel diagonals of orientation θ, we give an $O(n + R^3)$-time algorithm.

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