Crossing Minimization in Ortho-Radial Drawings of Column Trees ⋆

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As trees are widely used in representing hierarchical data in real-world applications, there has been a wide variety of drawing styles in visualizing trees. In some applications, nodes in a tree are partitioned into groups each containing those of similar nature, which leads to a drawing style with nodes in the same group residing in a *column* [1][1]. A *column tree* T is a triple $((V, E), h, c)$, where (V, E) is a rooted (ordered or unordered) tree, $h : V \to \mathbb{R}$ assigns each node $v \in V$ to its *height* $h(v)$, and $c : V \to \{1, ..., m\}$ maps each node v to a *column* $c(v)$. An edge is an *intra-edge* (resp., *inter-edge*) if the two ends of the edge lie in the same column (resp., different columns). Figure 1(a) displays the layout of a 3-column tree in an *orthogonal fashion*, in which each edge is a chain of horizontal and vertical segments with at most one bend. In drawing a column tree, the x-coordinate of a node can be adjusted to avoid intra-edge crossings, while the y-coordinate of a node is defined by h. The presence of inter-edges often makes edge crossings inevitable, as can be seen by the four crossings induced by inter-edge (b, c) in Figure 1(a). If one rotates the two subtrees rooted at a, the number of crossings is reduced to three. Furthermore, if in Column 1 the subtree rooted at d (which is a single node) is placed left to the one rooted at c, a crossing between edges (b, c) and (d, e) can be eliminated. It is therefore of interest to investigate the arrangement and the embedding of subtrees in each column to minimize the total number of crossings. Such an optimization issue is exactly what the work of [1] focuses on.

We extend prior study of column tree drawing in two directions: (1) using the style of *ortho-radial drawing*, and (2) allowing variable column order. An ortho-radial drawing is such that each edge is an alternating sequence of circular and radial segments on concentric circles. Figure 1(b) shows an ortho-radial drawing of the column tree in Figure 1(a). Clearly the freedom of allowing an edge to go either clockwise or counter-clockwise in ortho-radial drawing may provide a reduction in crossing number in comparison with its orthogonal counterpart. For example, changing the orientation of edge (b, c) from clockwise to counter-clockwise in Figure 1(b) reduces the number of crossings to one. Also, altering the column order from 1-2-3 to 2-1-3 in Figure 1(a) reduces the number of edge crossings cut by edge (b, c).

Given an m-column tree T and a number k, the *crossing minimization problem* (CMP, for short) is to decide whether there is an ortho-radial drawing of T with no more than k crossings. Note that the column number m is a problem parameter, unless stated otherwise.

Our main results include the following:

(Thm. 1) CMP is NP-complete for the following two cases:
- (a) T is an unordered binary tree with a fixed column order.
- (b) T is an ordered binary tree with a variable column order.

(Thm. 2) CMP is solvable in $O(2^d n^2)$ time when T is an unordered binary tree with a fixed column order, and d is the number of inter-edges.

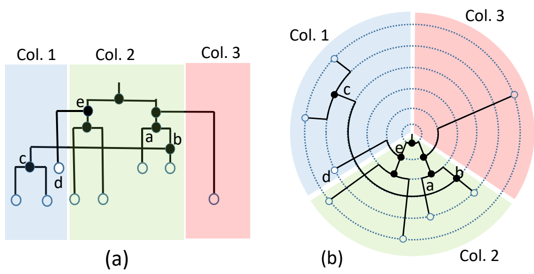

Fig. 1. (a): An orthogonal drawing; (b): An ortho-radial drawing.

The NP-hardness of Thm. 1(a) is proved by a reduction from Max-Cut, i.e., the problem of finding the maximum number of cut edges of a graph. The lower bound proof of Thm. 1(b) is by a reduction from the feedback arc set problem. It is interesting to point out that, in contrast to Thm. 1(a), CMP for unordered binary trees with a fixed column order under orthogonal drawing is solvable in $O(n^2)$ time [1]. The $O(2^d n^2)$ time algorithm in Thm. 2 takes advantage of a sweep-line algorithm proposed in [1]. To cope with the intractability result of Thm. 1, a natural attempt is to see whether good approximation solutions exist. As CMP under a variable column order is closely related to the well-known linear ordering problem (LOP), it would be interesting to see whether approximation algorithms for LOP could help in solving CMP.

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1 [1] Klawitter, J., Zink, J. Tree Drawings with Columns. Graph Drawing 2023, LNCS 14465.